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Isomorphisms between Leavitt algebras and their matrix rings

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Abstract. Let K be any field, let L_n denote the Leavitt algebra of type $(1, n-1)$ having coefficients in K, and let $M_d(L_n)$ denote the ring of $d \times d$ matrices over L_n. In our main result, we show that $M_d(L_n) \cong L_n$ if and only if d and $n-1$ are coprime. We use this isomorphism to answer a question posed in [14] regarding isomorphisms between various C^*-algebras. Furthermore, our result demonstrates that data about the K_0 structure is sufficient to distinguish up to isomorphism the algebras in an important class of purely infinite simple K-algebras.

Introduction

Let K be any field, and let $m < n$ be positive integers. The ring R is said to have *invariant basis number* (IBN) if no two free left R-modules of differing rank over R are isomorphic. On the other hand, R is said to have *module type* $(m, n-m)$ in case for every pair of positive integers a and b, (1) if $1 \leqq a < m$ then the free left R-modules R^a and R^i are not isomorphic for all positive integers $i \neq a$, and (2) if $a, b \geqq m$, then the free left R-modules R^a and R^b are isomorphic precisely when $a \equiv b \pmod{n-m}$. It is not hard to show that any non-IBN ring has module type $(m, n-m)$ for some pair of positive integers $m < n$. (The notation used here is not completely universal: some authors refer to the module type of such an algebra as the pair (m, n). Our notation is consistent with that used in many of the algebra articles on this topic, and is also consistent with the C^*-algebra usage as well.) As shown by Leavitt in [12], for every such pair m, n there exists a K-algebra $L_K(m, n)$ whose module type is $(m, n-m)$. In particular, the module type of $L_K(1, n)$ is $(1, n-1)$. We denote $L_K(1, n)$ by L_n. Various aspects of these algebras have been investigated, with

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