

# Math 2690

## Mathematical Problem Solving

Professor: Greg Marks

Fall 2017

Mon., 5:10–6:00 p.m.

216 Ritter Hall

1 unit

C.R.N.: 13057

This is a fun and interesting class for students at all levels who enjoy logical and mathematical puzzles and problems. Students will have the opportunity to:

- Learn various “tricks of the trade” not usually discussed in standard math classes—very useful for those going on to careers in high-tech industry or to graduate studies.
- Discover ways of applying what they have learned in diverse math courses to solve novel problems.
- Encounter math problems used in the real world as “entrance exams” by companies for prospective employees.
- Engage in mental exercise that should enhance their performance in all their classes.
- Participate in the William Lowell Putnam Mathematical Competition, a nationwide mathematics contest for undergraduates, which offers participants cash prizes of up to \$3,500, as well as prestige and accolades for the students and the university.

This course may be repeated for credit and will be graded on the Satisfactory/Unsatisfactory option. There will be no required homework, although students will be strongly encouraged to attempt the problems given out in class and participate in class discussions. Any student who participates in the Putnam Competition from 9 a.m. to 5 p.m. on the first Saturday in December will receive a grade of “Satisfactory” in this course, regardless of the score received in the Competition. There will be an emphasis throughout this course on past Putnam problems as preparation.

To illustrate the sort of challenging puzzle that is accessible to students at all levels, here is a sample problem:

*A famous riddle concerns a man with three sons who bequeaths half of his horses to his oldest son, one third of his horses to his middle son, and one ninth of his horses to his youngest son. Upon the man’s death, it transpires that he owns 17 horses, leaving the executor nonplussed about dividing them up. A neighbor comes to the rescue by lending his own horse to the estate; of the 18 horses, nine go to the oldest son, six to the middle son, and two to the youngest son, leaving one horse to be returned to the neighbor.*

*Now, here is the puzzle. A composer of riddles wants to create a problem of this form, but with different proportions in the man’s will. Thus the composer seeks positive integers  $a < b < c$  such that the oldest son is to receive  $1/a$ , the middle son  $1/b$ , and the youngest son  $1/c$  of the horses, and however many horses the composer decides that the man owns can be humanely distributed this way with one left over after the neighbor has contributed one additional horse. As indicated above,  $a = 2$ ,  $b = 3$ ,  $c = 9$  is one possibility. How many other choices of  $a$ ,  $b$ , and  $c$  are there?*