Math 2690
Mathematical Problem Solving

Professor: Greg Marks
Fall 2019
Mon., 5:10–6:00 p.m.
120 Ritter Hall
1 unit
C.R.N.: 11927

This is a fun and interesting class for students at all levels who enjoy logical and mathematical puzzles and problems. Students will have the opportunity to:

- Learn various "tricks of the trade" not usually discussed in standard math classes—very useful for those going on to careers in high-tech industry or to graduate studies.

- Discover ways of applying what they have learned in diverse math courses to solve novel problems.

- Encounter math problems used in the real world as "entrance exams" by companies for prospective employees.

- Engage in mental exercise that should enhance their performance in all their classes.

- Participate in the William Lowell Putnam Mathematical Competition, a nationwide mathematics contest for undergraduates, which offers participants cash prizes of up to $3,500, as well as prestige and accolades for the students and the university.

This course may be repeated for credit and will be graded on the Satisfactory/Unsatisfactory option. There will be no required homework, although students will be strongly encouraged to attempt the problems given out in class and participate in class discussions. Any student who participates in the Putnam Competition from 9 a.m. to 5 p.m. on the first Saturday in December will receive a grade of "Satisfactory" in this course, regardless of the score received in the Competition. There will be an emphasis throughout this course on past Putnam problems as preparation.

To illustrate the sort of challenging puzzle that is accessible to students at all levels, here is a sample problem:

A small Pacific island is populated by 1000 *people. Half of them speak Japanese; the other half speak Javanese. None of them can speak both languages. The island has a row of* 100 *old-fashioned phone booths, labeled* 1 *through* 100. *Every Saturday,* 100 *people on the island enter these phone booths, where they can make or receive phone calls, or simply relax quietly in the phone booth. Any calls between the phone booths must follow two rules:*

- *A phone call may occur only between two speakers of the same language.*

- *On a given Saturday, a phone booth labeled with a certain number cannot make more than one connection with the set of phone booths labeled with larger numbers, irrespective of which side originates the call. (There is no restriction on the number of phone calls between a phone booth and lower-numbered phone booths that have not yet reached their own one-call limits.)*

For example, it would be consistent with these rules if there were a phone conversation between Javanese speakers in phone booths #4 and #90, a phone conversation between Javanese speakers in phone booths #73 and #90, a phone conversation between Japanese speakers in phone booths #26 and #84, a phone conversation between Japanese speakers in phone booths #51 and #84, and a phone conversation between Japanese speakers in phone booths #84 and #97, while the people in the other 93 phone booths just sat there without calling or being called by anyone.

At the end of the day Saturday, the island phone commissioner compiles a record consisting of the language spoken by the person in each of the 100 *phone booths, plus a list of the pairs of phone booths between which a conversation occurred. (For each phone conversation, no record is kept of which phone booth placed the call and which received the call.)*

*How many different records are possible? (**Hint.** The answer is greater than* 2^{100}, *since that is the number of records in which no phone calls have been placed.)*